The (e-p-v) Equation of State

An experimental Hugoniot curve $p_H(p_o,v_o,v)$ is the locus of experimentally measured pressure-volume states produced by passing constant velocity shocks of various strengths into an initial state (p_o,v_o). The change in internal energy along a Hugoniot curve is given by Eq. 4. A family of Hugoniot curves, each of which is centered on a curve along which the energy change is known, is therefore sufficient to determine the (e-p-v) equation of state over the domain of the (p-v) plane covered by the Hugoniots.

In the present work we choose to measure a family of Hugoniots centered on the atmospheric ($p \approx 0$) isobar, because the energy change can easily be measured along this cross curve.

Calculation of the (T-p-v) from the (e-p-v) Equation of State

Since the (T-p-v) and (e-p-v) equations of state are both incomplete, it is necessary to establish what additional data are required to calculate temperature when the (e-p-v) relationship is known. It follows from thermodynamic identities that the (e-p-v) and (T-p-v) equations of state are related through isentropes. The position of an isentrope in the (p-v) plane is determined by the (e-p-v) equation of state and the isentropic condition $de = -pdv$ obtained by setting $ds = 0$ in Eq. 6. The temperature along an isentrope is given as

$$T = T_i \exp\left[- \int_{v_i}^{v} \left(\frac{\partial p}{\partial e}\right)_v dv \right] \qquad (9)$$

by integrating the identity

$$ds = \left(\frac{\partial e}{\partial T}\right)_v \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dv \qquad (10)$$

subject to the isentropic condition $ds = 0$.

Thus to calculate the (T-p-v) from the (e-p-v) equation of state, it is necessary first to construct a family of isentropes in the (p-v) plane and then to calculate temperature along them with Eq. 9. It is important to note that temperature cannot be calculated with Eq. 9 unless the temperatures T_i at particular points (p_i, v_i) on the isentropes are known. Measurement of the temperature along any curve which intersects the entire family of isentropes permits a value of T_i to be assigned to each isentrope. Thus the (T-p-v) equation of state is determined in the domain of the (p-v) plane covered by a family of isentropes. For a given (e-p-v) equation of state, there is a thermodynamically consistent (T-p-v) equation of state for each assignment of temperature along a nonisentropic curve.

Measurements of temperature and energy along the atmospheric isobar are sufficient to calculate the (T-p-v) and (e-p-v) equations of state from a family of Hugoniot curves centered on this isobar. However, the (e-p-v) and (T-p-v) equations of state will necessarily be specified over different but overlapping domains of the (p-v) plane. The family of Hugoniot curves defines the domain where the (e-p-v) equation of state is known, but the family of isentropes constructed from the (e-p-v) relationship defines the subdomain where the (T-p-v) equation of state is known.

III. EXPERIMENTS

Dow Corning silicone 210 fluid (100 centistokes) was chosen because of its good thermal stability and its large coefficient of expansion. Static experiments were performed to measure the variation of density and specific enthalpy h, along the atmospheric isobar. Shock wave experiments were performed to determine a family of Hugoniot curves centered on the atmospheric isobar.

Static Measurements

The variation of volume with temperature at atmospheric pressure between $-30^{o}C$ and $150^{o}C$ was measured with a density balance. A least squares fit for the data, with T in degrees Kelvin,

$$v^{-1} = 1.2566 + 1.0577 \times 10^{-3} T + 2.604 \times 10^{-7} T^2$$